NATIONAL UNIVERSITY OF SAN MARCOS

FACULTY OF MATHEMATICAL SCIENCES

SYLLABUS

1. GENERAL INFORMATION

1.1 Course name	:	Functional analysis I
1.2 Code	:	MA371
1.3 Academic Program	:	Scientific Computing
1.4 Credits	:	5
1.5 Prerrequisites	:	• MA222. Real Analysis I. (5^{th} Sem) • ID201. Technical and professional English III. (5^{th} Sem) • MA122. Linear Algebra II. (4^{th} Sem)
1.6 Type of Course	:	Mandatory
1.7 Area of Studies	:	Estudios Generales
1.8 Number of Weeks	:	16
1.9 Hours per week	:	3 HT; 4 HP;
1.10 Cycle	:	I
1.11 Learning modality	:	Face to face
1.12 Academic Period	:	2026-I
1.13 Semester Start Date	:	
1.14 Semester End Date	:	

2. PROFESSORS

Meetings after coordination with the professor

3. INTRODUCTION TO THE COURSE

Functional analysis is a central pillar of modern mathematics, providing the framework to solve problems in differential equations, quantum mechanics, and numerical analysis. This course bridges the gap between linear algebra and infinite-dimensional spaces. By introducing Measure Theory as a rigorous foundation for integration, the course enables the study of Banach and Hilbert spaces, linear operators, and the fundamental theorems that govern functional systems.

4. GOALS

- Establish a rigorous understanding of Lebesgue measure and integration theory.
- Master the topological and algebraic structures of Banach and Hilbert spaces.
- Apply the fundamental theorems of functional analysis (Hahn-Banach, Open Mapping, Closed Graph).
- Analyze the properties of linear operators and their spectral theory in infinite dimensions.

5. COMPETENCES

ABET-1) An ability to identify, formulate, and solve broadly defined technical or scientific problems by applying knowledge of mathematics and science and/or technical topics to areas relevant to the discipline. (Usage)

6. TOPICS

Unit 1: Measure and Integration (16 hours)	
Competences Expected: ABET-1	
Topics	**Learning Outcomes**
• σ-algebras and measurable spaces • Lebesgue measure on $\mathbb{R}^n$ • Measurable functions and Lebesgue integration • Convergence theorems (Monotone, Dominated, Fatou's Lemma) • L^p spaces: completeness and duality	• Construct the Lebesgue measure and identify measurable sets [Assessment] • Evaluate integrals using convergence theorems [Usage] • Prove the completeness of L^p spaces [Familiarity]
Readings : [**Folland1999**], [**Royden2010**]	

Unit 2: Normed and Banach Spaces (16 hours)	
Competences Expected: ABET-1	
Topics	**Learning Outcomes**
• Normed vector spaces and completeness • Schauder bases and infinite dimensionality • Linear operators and functionals • Dual spaces and the Hahn-Banach theorem • Category theorems: Baire, Open Mapping, and Closed Graph	• Verify the completeness of various functional spaces [Assessment] • Apply the Hahn-Banach theorem to extend linear functionals [Usage] • Utilize the Open Mapping theorem to determine operator boundedness [Assessment]
Readings : [**Kreyszig1989**], [**Brezis2011**]	

Unit 3: Hilbert Spaces (16 hours)	
Competences Expected: ABET-1	
Topics	**Learning Outcomes**
• Inner product spaces and Hilbert structure • Orthogonality, projection theorem, and Riesz representation • Orthonormal sets and Fourier series in Hilbert spaces • Self-adjoint, unitary, and normal operators	• Construct orthogonal projections in Hilbert spaces [Usage] • Represent linear functionals using the Riesz theorem [Assessment] • Analyze the geometry of infinite-dimensional inner product spaces [Familiarity]
Readings : [**Kreyszig1989**], [**Conway1990**]	

Unit 4: Operator Theory and Spectral Theory (16 hours)	
Competences Expected: ABET-1	
Topics	**Learning Outcomes**
• Compact operators on Banach and Hilbert spaces • The Fredholm alternative • Spectral theory of bounded linear operators • Spectral theorem for compact self-adjoint operators	• Identify compact operators and their properties [Assessment] • Determine the spectrum of linear operators [Usage] • Apply the spectral theorem to decompose operators [Assessment]
Readings : [**Brezis2011**], [**Yosida1980**]	

7. WORKPLAN

7.1 Methodology

Individual and team participation is encouraged to present their ideas, motivating them with additional points in the different stages of the course evaluation.

7.2 Theory Sessions

The theory sessions are held in master classes with activities including active learning and roleplay to allow students to internalize the concepts.

7.3 Practical Sessions

The practical sessions are held in class where a series of exercises and/or practical concepts are developed through problem solving, problem solving, specific exercises and/or in application contexts.

8. EVALUATION SYSTEM

********* EVALUATION MISSING ********

9. BASIC BIBLIOGRAPHY