NATIONAL UNIVERSITY OF SAN MARCOS

FACULTY OF MATHEMATICAL SCIENCES

SYLLABUS

1. GENERAL INFORMATION

1.1 Course name	:	Functional analysis II
1.2 Code	:	MA372
1.3 Academic Program	:	Scientific Computing
1.4 Credits	:	5
1.5 Prerrequisites	:	MA371. Functional analysis I. (6^{th} Sem)
1.6 Type of Course	:	Elective
1.7 Area of Studies	:	Estudios Generales
1.8 Number of Weeks	:	16
1.9 Hours per week	:	3 HT; 4 HP;
1.10 Cycle	:	I
1.11 Learning modality	:	Face to face
1.12 Academic Period	:	2026-I
1.13 Semester Start Date	:	
1.14 Semester End Date	:	

2. PROFESSORS

Meetings after coordination with the professor

3. INTRODUCTION TO THE COURSE

This course deepens the theoretical foundations of measure and integration, providing essential tools for advanced analysis in infinite-dimensional spaces. It focuses on the rigorous construction of abstract measures and the operational aspects of the Lebesgue integral. Finally, the course explores the geometric structure and operator theory in Hilbert spaces, which are fundamental for developing computational methods and models in mathematical physics.

4. GOALS

- Understand and construct abstract measures and measure extensions in general spaces.
- Master Lebesgue integration theory and its fundamental convergence theorems.
- Analyze measure duality and decomposition through the Radon-Nikodym theorem.
- Study advanced geometry and operator theory within Hilbert spaces.

5. COMPETENCES

ABET-1) An ability to identify, formulate, and solve broadly defined technical or scientific problems by applying knowledge of mathematics and science and/or technical topics to areas relevant to the discipline. (Usage)

ABET-3) An ability to develop and conduct experiments or test hypotheses, analyze and interpret data, and use scientific judgment to draw conclusions. (Usage)

6. TOPICS

Unit 1: Measure Theory (20 hours)	
Competences Expected: ABET-1,ABET-3	
Topics	**Learning Outcomes**
• Algebras, σ-algebras, and measures • Outer measures and the Carathéodory Extension Theorem • Borel measures on the real line • Signed measures and the Hahn-Jordan decomposition • Complex measures and total variation	• Construct measures from outer measures using the Carathéodory criterion [Assessment] • Apply the Jordan decomposition to analyze signed measures [Usage] • Prove regularity properties of Borel measures [Familiarity]
Readings : [**Folland1999**], [**Royden2010**]	

Unit 2: Integration Theory (20 hours)	
Competences Expected: ABET-1,ABET-3	
Topics	**Learning Outcomes**
• Measurable functions and simple functions • The Lebesgue integral: definition and fundamental properties • Convergence theorems: Monotone Convergence, Dominated Convergence, and Fatou's Lemma • The Radon-Nikodym Theorem and Lebesgue-Stieltjes property • Differentiation of measures and the Fundamental Theorem of Calculus in measure spaces	• Evaluate the convergence of function sequences under the Lebesgue integral [Assessment] • Use the Radon-Nikodym Theorem to find probability densities or measure derivatives [Usage] • Compare Riemann and Lebesgue integrals in terms of applicability [Familiarity]
Readings : [**Folland1999**], [**Brezis2011**]	

Unit 3: Hilbert Spaces (20 hours)	
Competences Expected: ABET-1,ABET-3	
Topics	**Learning Outcomes**
• Review of inner products and completeness • Orthogonality, orthonormal bases, and the Gram-Schmidt process • Riesz Representation Theorem and linear functionals • Adjoint, self-adjoint, and unitary operators in Hilbert spaces • Introduction to spectral theory for compact operators	• Apply the projection theorem to solve best approximation problems [Usage] • Identify the adjoint of classical linear operators [Assessment] • Analyze the spectral structure of self-adjoint operators [Familiarity]
Readings : [**Kreyszig1989**], [**Conway1990**]	

Unit 4: Operator Theory (20 hours)	
Competences Expected: ABET-1,ABET-3	
Topics	**Learning Outcomes**
• Bounded and unbounded operators: definitions and domains • The spectrum of an operator: point, continuous, and residual spectrum • Normal operators and the Spectral Theorem for compact operators • Applications of operator theory in numerical analysis	• Calculate the spectrum of various types of linear operators [Assessment] • Apply the spectral theorem in operator decomposition [Usage] • Solve operational problems using basic functional calculus [Assessment]
Readings : [**Brezis2011**], [**Yosida1980**]	

7. WORKPLAN

7.1 Methodology

Individual and team participation is encouraged to present their ideas, motivating them with additional points in the different stages of the course evaluation.

7.2 Theory Sessions

The theory sessions are held in master classes with activities including active learning and roleplay to allow students to internalize the concepts.

7.3 Practical Sessions

The practical sessions are held in class where a series of exercises and/or practical concepts are developed through problem solving, problem solving, specific exercises and/or in application contexts.

8. EVALUATION SYSTEM

********* EVALUATION MISSING ********

9. BASIC BIBLIOGRAPHY