NATIONAL UNIVERSITY OF SAN MARCOS

FACULTY OF MATHEMATICAL SCIENCES

SYLLABUS

1. GENERAL INFORMATION

1.1 Course name	:	Algebraic Structures
1.2 Code	:	MA381
1.3 Academic Program	:	Scientific Computing
1.4 Credits	:	4
1.5 Prerrequisites	:	MA281. Numbers theory. (5^{th} Sem)
1.6 Type of Course	:	Elective
1.7 Area of Studies	:	Estudios Generales
1.8 Number of Weeks	:	16
1.9 Hours per week	:	2 HT; 4 HP;
1.10 Cycle	:	I
1.11 Learning modality	:	Face to face
1.12 Academic Period	:	2026-I
1.13 Semester Start Date	:	
1.14 Semester End Date	:	

2. PROFESSORS

Meetings after coordination with the professor

3. INTRODUCTION TO THE COURSE

Algebraic Structures provides the formal and abstract foundation essential for advanced mathematics and its computational applications. This course introduces the fundamental structures of modern algebra—groups, rings, and fields—developing the ability to think in terms of axioms, homomorphisms, and invariants. Mastery of these concepts is crucial for understanding the theoretical underpinnings of areas such as coding theory, cryptography, algorithm design, and algebraic geometry. The course emphasizes the interplay between abstract theory and concrete examples, particularly those arising in computational and discrete mathematics.

4. GOALS

- Master the axiomatic definitions and basic properties of groups, rings, and fields.
- Understand and apply the concepts of subgroups, normal subgroups, quotient groups, and the fundamental homomorphism theorems for groups.
- Analyze the structure of rings, including ideals, quotient rings, integral domains, and fields.
- Apply the theory of finite fields to practical problems in coding theory and cryptography.
- Develop the ability to construct rigorous proofs and recognize algebraic structures in various mathematical and computational contexts.

5. COMPETENCES

ABET-1) An ability to identify, formulate, and solve broadly defined technical or scientific problems by applying knowledge of mathematics and science and/or technical topics to areas relevant to the discipline. (Usage)

ABET-3) An ability to develop and conduct experiments or test hypotheses, analyze and interpret data, and use scientific judgment to draw conclusions. (Usage)

6. TOPICS

Unit 1: Groups and Subgroups (16 hours)	
Competences Expected: ABET-1,ABET-3	
Topics	**Learning Outcomes**
• Binary operations and the definition of a group. Basic examples: cyclic, symmetric, dihedral, and matrix groups. • Elementary properties of groups: uniqueness of identity and inverses, cancellation laws. • Subgroups and subgroup tests. Centralizer, normalizer, and center of a group. • Cyclic groups: classification, structure, and subgroups. • Permutation groups: cycle notation, transpositions, and parity.	• State the group axioms and verify whether a given set with an operation forms a group [Familiarizarse]. • Apply subgroup tests and generate subgroups, including cyclic subgroups of a given element [Usar]. • Analyze the structure of finite cyclic groups and decompose permutations into disjoint cycles [Evaluar].
Readings : [**Dummit2004**], [**Gallian2021**]	

Unit 2: Cosets, Normal Subgroups, and Homomorphisms (16 hours)	
Competences Expected: ABET-1,ABET-3	
Topics	**Learning Outcomes**
• Cosets and Lagrange's Theorem. Applications to the order of elements. • Normal subgroups and their characterization. Quotient (factor) groups. • Group homomorphisms and isomorphisms: definitions, properties, and examples. • The Fundamental Homomorphism Theorem (First Isomorphism Theorem) for groups. • Introduction to group actions: definition and basic examples (orbit-stabilizer).	• Compute cosets and use Lagrange's Theorem to deduce properties of finite groups [Familiarizarse]. • Determine if a subgroup is normal and construct the corresponding quotient group [Usar]. • Use the Fundamental Homomorphism Theorem to establish isomorphisms between quotient groups and known groups [Evaluar].
Readings : [**Dummit2004**], [**Judson2021**]	

Unit 3: Rings, Ideals, and Integral Domains (16 hours)	
Competences Expected: ABET-1,ABET-3	
Topics	**Learning Outcomes**
• Definition and basic examples of rings, commutative rings, and rings with unity. • Ring homomorphisms, isomorphisms, and basic properties. • Ideals and quotient rings. The Fundamental Homomorphism Theorem for rings. • Operations on ideals: sum, product, and intersection. • Integral domains, fields, and characteristic of a ring. Finite integral domains are fields.	• Identify the defining properties of rings, integral domains, and fields from given examples [Familiarizarse]. • Verify whether a subset of a ring is an ideal and construct the corresponding quotient ring [Usar]. • Apply ring homomorphism theorems to analyze ring structures and prove basic results about integral domains [Evaluar].
Readings : [**Dummit2004**], [**Hungerford2014**]	

Unit 4: Fields and Applications (16 hours)	
Competences Expected: ABET-1,ABET-3	
Topics	**Learning Outcomes**
• Field extensions: algebraic and transcendental elements. Degree of an extension. • Finite fields (Galois fields): existence, uniqueness, and structure. The field $GF(p^n)$. • Polynomial rings over a field. Irreducible polynomials and construction of finite fields. • Primitive elements in finite fields and their application in cryptography (e.g., Diffie-Hellman, AES). • Brief introduction to error-correcting codes: Hamming distance and linear codes as vector spaces over finite fields.	• Describe the classification and basic properties of finite fields [Familiarizarse]. • Construct small finite fields using irreducible polynomials over $\mathbb{Z}_p$ [Usar]. • Explain how the structure of finite fields underpins modern cryptographic protocols and coding theory [Evaluar].
Readings : [**Gallian2021**], [**Lidl1994**]	

7. WORKPLAN

7.1 Methodology

Individual and team participation is encouraged to present their ideas, motivating them with additional points in the different stages of the course evaluation.

7.2 Theory Sessions

The theory sessions are held in master classes with activities including active learning and roleplay to allow students to internalize the concepts.

7.3 Practical Sessions

The practical sessions are held in class where a series of exercises and/or practical concepts are developed through problem solving, problem solving, specific exercises and/or in application contexts.

8. EVALUATION SYSTEM

********* EVALUATION MISSING ********

9. BASIC BIBLIOGRAPHY